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CP Violation in the Neutral *K* System†

(A brief review of the theory)

SUK KOO YUN‡

Department of Physics, Syracuse University, Syracuse, New York 13210

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Abstract

The theoretical aspect of *CP*-violation is reviewed. The K_S and K_L states, the unitarity relation, parametrization and analysis, the electric dipole moment of the neutron, η asymmetry and *C* noninvariance in $\eta_{3\pi}$ are all discussed. Then five theoretical models concerning neutral $K \to 2\pi$ are dealt with.

1. *Introduction*

Although the possibility of *CP* violations was previously discussed theoretically, the discovery of the $K_L \to 2\pi$ decay (i.e. $K_L \to \pi^+\pi^-$) by Christensen *et al.* (1964) started the discussions of the topic in earnest.

Experimentally (Barash-Schmidt *et al.*, 1969) we observe the following nonleptonic weak decays of the neutral *K* system:

$$\text{The neutral } K \text{ system} \to \begin{cases} \left.\begin{array}{l} \pi^+\pi^- \ (68{\cdot}1 \pm 1{\cdot}1)\% \\ \pi^0\pi^0 \ (31{\cdot}6 \pm 1{\cdot}1)\% \end{array}\right\} \tau_S = 0{\cdot}874 \times 10^{-10} \text{ sec} \\ \left.\begin{array}{l} \pi^0\pi^0 \ {}^0(2{\cdot}5 \pm 0{\cdot}7)\% \\ \pi^+\pi^- \ {}^0(28{\cdot}1 \pm 0{\cdot}8)\% \\ \pi^+\pi^- \ (0{\cdot}157 \pm 0{\cdot}00+)\% \\ \pi^0\pi^0 \ (\text{uncertain}) \end{array}\right\} \tau_L = 5{\cdot}30 \times 10^{-8} \text{ sec} \end{cases} \tag{1.1}$$

Here we notice the two definitely distinct mean lives τ_S and τ_L for the neutral *K* system.

According to the *CPT* theorem (or Lüder's theorem), 'If a theory of interacting fields obeys the Wightman postulates and is invariant under the restricted Lorentz group (i.e. without any discrete element like *P* or *T*) then it will be invariant under *CPT*'. If we assume *CPT* invariance (which we shall do throughout this review), a consequence of the theorem

† Work supported in part by the United States Atomic Energy Commission.

‡ Present address: Department of Physics, Saginaw Valley College, University Center, Michigan 48710.

is that the observable mass and lifetime of a particle are always exactly the same as those of the corresponding antiparticle.

Thus the parent states of the two different lifetimes τ_S and τ_L cannot be K^0 or $\overline{K^0}$. Note that since the system is neutral its decay is indirectly observed. We measure the energy-momentum of the decay product and measure the quantum numbers of the products system instead of the neutral K system. Therefore, we name the short-lived and long-lived parent states as K_S and K_L, respectively, and have

$$\begin{array}{ll} & \qquad\quad CP \\ K_S \rightarrow \begin{cases} \pi^+\pi^-, & +1 \\ \pi^0\pi^0, & +1 \end{cases} & \\ K_L \rightarrow \begin{cases} \pi^0\pi^0\pi^0, & -1 \\ \pi^+\pi^-\pi^0, & -1 \\ \pi^+\pi^-, & +1 \\ \pi^0\pi^0, & +1 \end{cases} & \end{array} \tag{1.2}$$

Here we have used the following conventions:

$$\begin{aligned} C|K^0\rangle &= |\overline{K^0}\rangle \\ CP|K^0\rangle &= -|\overline{K^0}\rangle \\ CP|\pi^0\rangle &= -|\pi^0\rangle \\ CP|\pi^+\pi^-\rangle &= +|\pi^+\pi^-\rangle \text{ in the center-of-mass system.} \end{aligned} \tag{1.3}$$

In the decay modes shown in (1.2), if $K_L \not\rightarrow 2\pi$ then the K_S and K_L states may be assigned $CP = +1$ and -1, respectively, and CP conservation holds. But the K_L decays show a clear CP violation.

2. K_S and K_L States

Now we can ask the following question: What are the K_S and K_L states?† In strong interactions (H_S), the hypercharge Y is conserved, i.e. $\Delta Y = 0$, and $|K^0\rangle$ and $|\overline{K^0}\rangle$ are eigenstates of H_S. Therefore, transitions $K^0 \leftrightarrow \overline{K^0}$ are not allowed. Thus the two particles K^0 and $\overline{K^0}$ are quite distinct, and we can tell definitely which of the two particles is produced. Since weak interactions (H_W) do not conserve hypercharge, as soon as the weak interaction is turned on, the hypercharge is no longer a good quantum number (i.e. $\Delta Y \neq 0$). Thus the following is possible:

$$K^0 \rightarrow \pi^+\pi^- \rightarrow \overline{K^0}$$

and K^0 and $\overline{K^0}$ become degenerate. $|K^0\rangle$ and $|\overline{K^0}\rangle$ are no longer different states, but the combinations of the two states may be the eigenstates of the mass operator.

† For the discussions of K_S and K_L states, see, for example, Gasiorowicz, S. (1966). *Elementary Particle Physics*. John Wiley, New York.

We consider the S-matrix acting in the space spanned by $|K^0\rangle$ and $|\overline{K^0}\rangle$ states. To discuss the $K^0 - \overline{K^0}$ mixing, we have to work up to the second order in H_W, because of the fact that $\langle K^0|H_W|K^0\rangle = 0$. In the $K^0 - \overline{K^0}$ space we can form a 2×2 matrix,

$$\begin{pmatrix} \langle K^0|T|K^0\rangle, \langle K^0|T|\overline{K^0}\rangle \\ \langle \overline{K^0}|T|K^0\rangle, \langle \overline{K^0}|T|\overline{K^0}\rangle \end{pmatrix} \equiv \begin{pmatrix} a & b \\ c & a \end{pmatrix} \tag{2.1}$$

where

$$T = H_W + H_W \frac{1}{E - H_S + i\epsilon} H_W$$

and by CPT

$$\langle K^0|T|K^0\rangle = \langle \overline{K^0}|T|\overline{K^0}\rangle \equiv a \tag{2.2}$$

Note that in equation (2.1) b and c have a mass part and a decay part.

We now diagonalize equation (2.1) and find the mixing of $|K^0\rangle$ and $|\overline{K^0}\rangle$. These mixing states will be denoted by $|K_S\rangle$ and $|K_L\rangle$, each having its own decay rate. Thus there is no transition between $|K_S\rangle$ and $|K_L\rangle$,

$$\langle K_S|T|K_L\rangle = \langle K_L|T|K_S\rangle = 0 \tag{2.3}$$

By the diagonalization procedure we get the eigenvalues

$$\lambda_{\substack{S\\L}} = a \pm \sqrt{bc} \tag{2.4}$$

and the eigenvectors corresponding to these are

$$\begin{aligned} |K_S\rangle &= p|K^0\rangle - q|\overline{K^0}\rangle \\ |K_L\rangle &= p|K^0\rangle + q|\overline{K^0}\rangle \end{aligned} \tag{2.5}$$

where

$$\begin{aligned} |p|^2 &= \frac{b}{b+c} \\ |q|^2 &= \frac{c}{b+c} \\ |p|^2 + |q|^2 &= 1 \end{aligned} \tag{2.6}$$

Equation (2.5) is found under the assumption that only CPT invariance holds. However, if CP is also an invariance, then

$$\langle \overline{K^0}|T|K^0\rangle = \langle K^0|T|\overline{K^0}\rangle$$

i.e.,

$$b = c \tag{2.7}$$

and

$$|p|^2 = |q|^2 = \tfrac{1}{2} \tag{2.8}$$

by equation (2.6).

Then we can choose the CP-even and CP-odd eigenstates as

$$|K_1\rangle = \frac{-i}{\sqrt{2}}(|K^0\rangle - |\overline{K^0}\rangle) \tag{2.9}$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K^0}\rangle)$$

where

$$\begin{aligned} CP|K_1\rangle &= |K_1\rangle \\ CP|K_2\rangle &= -|K_2\rangle \end{aligned} \tag{2.10}$$

We now consider the CP-nonconserving overlap α defined as

$$\alpha \equiv \langle K_L | K_S \rangle \tag{2.11}$$

If CP is violated, the CP-nonconserving overlap is, in general,

$$\alpha \equiv \langle K_L | K_S \rangle = |p|^2 - |q|^2 \neq 0 \tag{2.12}$$

by equations (2.5) and (2.6). However, if CP is conserved,

$$\alpha \equiv \langle K_L | K_S \rangle = \langle K_2 | K_1 \rangle = 0 \tag{2.13}$$

by equations (2.8) and (2.12), i.e. the K_1 and K_2 are orthogonal when CP is a good quantum number.

3. *Unitarity Relation*

We assume the exponential decay in deriving the unitarity relation.† Since the Lorentz invariance allows us to use the rest frame, we consider the time evolution of states in terms of proper time t,

$$\begin{aligned} |K_S\rangle &\to \exp(-iM_S t)|K_S\rangle \\ |K_L\rangle &\to \exp(-iM_L t)|K_L\rangle \end{aligned} \tag{3.1}$$

where M_S and M_L are, respectively, related to the masses m_S and m_L and to the decay rates Γ_S and Γ_L by

$$\begin{aligned} M_S &= m_S - \frac{i}{2}\Gamma_S \\ M_L &= m_L - \frac{i}{2}\Gamma_L \end{aligned} \tag{3.2}$$

We denote the general state of the neutral K system by Ψ, and have

$$\Psi = A_S \exp(-iM_S t)|K_S\rangle + A_L \exp(-iM_L t)|K_L\rangle \tag{3.3}$$

† For the discussions of unitarity relations, see, for example, Bell, J. S. and Steinberger, J. (1965). 'Weak Interactions of Kaons'. Lectures given at the Oxford International Conference on Elementary Particles, September, 1965.

The norm then can be written as

$$
\begin{aligned}
|\Psi|^2 = |A_S|^2 \exp(-\Gamma_S t) + |A_L|^2 \exp(-\Gamma_L t) \\
+ A_S A_L^* \exp[i(M_L^* - M_S) t] \langle K_L | K_S \rangle \\
+ A_L A_S^* \exp[i(M_S^* - M_L) t] \langle K_S | K_L \rangle
\end{aligned} \tag{3.4}
$$

At $t = 0$, we get

$$
\begin{aligned}
-\frac{d|\Psi|^2}{dt}\bigg|_{t=0} = \Gamma_S |A_S|^2 + \Gamma_L |A_L|^2 - i(M_L^* - M_S) A_S A_L^* \langle K_L | K_S \rangle \\
- i(M_S^* - M_L) A_L A_S^* \langle K_S | K_L \rangle
\end{aligned} \tag{3.5}
$$

On the other hand, we can write down the total transition rate as

$$
\text{Total transition rate} = \sum_f |A_S \langle f | T | K_S \rangle + A_L | \langle f | T | K_L \rangle|^2 \tag{3.6}
$$

for arbitrary A_S and A_L, and summation is over all final states.

Since equations (3.5) and (3.6) are supposed to be the same, by comparing the two equations we get

$$
\begin{aligned}
\Gamma_S = \sum_f |\langle f | T | K_S \rangle|^2 \\
\Gamma_L = \sum_f |\langle f | T | K_L \rangle|^2 \\
-i(M_L^* - M_S) \langle K_L | K_S \rangle = \sum_f \langle f | T | K_L \rangle^* \langle f | T | K_S \rangle
\end{aligned} \tag{3.7}
$$

This is the unitarity relation for the neutral *K* system.

4. *Parametrization and Analysis*

Since the final 2π states in the neutral $K \to 2\pi$ decays are symmetric, the allowed final isospin states are $I_f = 0$ and 2 for the *S*-waves. Thus there are four amplitudes in the $K_S \to 2\pi$ and $K_L \to 2\pi$ decays. They are

$$
\begin{aligned}
\langle I_f = 0 | T | K_S \rangle, \qquad \langle I_f = 2 | T | K_S \rangle \\
\langle I_f = 0 | T | K_L \rangle, \qquad \langle I_f = 2 | T | K_L \rangle
\end{aligned} \tag{4.1}
$$

In equations (4.1) the amplitudes with $I_f = 0$ are for the $\Delta I = \frac{1}{2}$ transitions, and the ones with $I_f = 2$ are for the $\Delta I = \frac{3}{2}$ transitions.

In order to establish the relative magnitudes and the phases of these amplitudes, we define the amplitude ratios†

$$
\epsilon \equiv \frac{\langle 0 | T | K_L \rangle}{\langle 0 | T | K_S \rangle} \tag{4.2a}
$$

$$
\epsilon' \equiv \frac{1}{\sqrt{2}} \frac{\langle 2 | T | K_S \rangle}{\langle 0 | T | K_S \rangle} \tag{4.2b}
$$

† For the discussions of parametrization, see Wu, T. T. and Yang, C. N. (1964). *Physical Review Letters*, **13**, 380. See also, the reference quoted in the footnote to p. 370.

$$\omega \equiv \frac{1}{\sqrt{2}} \frac{\langle 2|T|K_S\rangle}{\langle 0|T|K_S\rangle} \tag{4.2c}$$

To relate the theoretical parameters defined in equations (4.2a)–(4.2c) to the experimentally available information, we define the following experimental parameters:

$$\eta_{+-} \equiv |\eta_{+-}| \exp(i\phi_{+-}) \equiv \frac{\langle \pi^+\pi^-|T|K_L\rangle}{\langle \pi^+\pi^-|T|K_S\rangle} \tag{4.3a}$$

$$\eta_{00} \equiv |\eta_{00}| \exp(i\phi_{00}) \equiv \frac{\langle \pi^0\pi^0|T|K_L\rangle}{\langle \pi^0\pi^0|T|K_S\rangle} \tag{4.3b}$$

$$R \equiv \frac{\Gamma(K_S \to \pi^0\pi^0)}{\Gamma(K_S \to \pi^+\pi^-)} = \text{real quantity} \tag{4.3c}$$

In addition to these we have the unitarity relation equation (3.7), which may be written as

$$-i(M_L{}^* - M_S)\langle K_L|K_S\rangle = \{\langle 0|T|K_L\rangle^* \langle 0|T|K_S\rangle + \langle 2|T|K_L\rangle^* \langle 2|T|K_S\rangle\} \tag{4.3d}$$

The real part of equation (4.3d) is a trivial identity $\Gamma_S = \Gamma_S$. So this equation gives only one useful relation.

The two-pion states can be expressed in terms of the isospin states

$$|\pi^+\pi^-\rangle = \frac{1}{\sqrt{6}}(\sqrt{2}|0\rangle + |2\rangle) \tag{4.4a}$$

$$|\pi^0\pi^0\rangle = -\frac{\sqrt{6}}{3}\left(\frac{1}{\sqrt{2}}|0\rangle - |2\rangle\right) \tag{4.4b}$$

With the aid of equations (4.4a) and (4.4b), the experimental parameters may be expressed in terms of the theoretical parameters, as follows:

$$\eta_{+-} = \frac{\epsilon + \epsilon'}{1 + \omega} \tag{4.5a}$$

$$\eta_{00} = \frac{\epsilon - 2\epsilon'}{1 - 2\omega} \tag{4.5b}$$

$$R = \frac{\frac{1}{2!}|\langle \pi^0\pi^0|T|K_S\rangle|^2}{|\langle \pi^+\pi^-|T|K_S\rangle|^2} = \frac{1}{2}\frac{|1 - 2\omega|^2}{|1 + \omega|^2} \tag{4.5c}$$

Experimentally $|\omega|$ is the order 10^{-2}. Thus η_{+-} and η_{00} can be expanded,

$$\eta_{+-} \simeq \epsilon - (\epsilon' - \epsilon\omega) - \epsilon'\omega \tag{4.6a}$$

$$\eta_{00} \simeq \epsilon - 2(\epsilon' - \epsilon\omega) - 4\epsilon'\omega \tag{4.6b}$$

There is a possibility that $|\epsilon'|$ and $|\epsilon\omega|$ are of the same order. If that is true, the last terms in equations (4.6a) and (4.6b) may not be neglected.

If we neglect ω we get

$$\eta_{+-} \simeq \epsilon + \epsilon' \tag{4.7a}$$

$$\eta_{00} \simeq \epsilon - 2\epsilon' \tag{4.7b}$$

which in turn gives the relation

$$2\eta_{+-} + \eta_{00} \simeq 3\epsilon \tag{4.7c}$$

Furthermore, if we neglect $\Delta I = \frac{3}{2}$ transitions, then we get

$$\eta_{+-} = \eta_{00} = \epsilon \tag{4.8}$$

When final state interactions are neglected, then, on account of *CPT* invariance,

$$\langle f|T|\overline{K^0}\rangle = \langle f'|T|K^0\rangle^* \tag{4.9}$$

provided T is Hermitian. If we include the final state interactions, by Watson's Theorem equation (4.9) changes to

$$\langle f|T|\overline{K^0}\rangle = \exp(2i\delta)\langle \bar{f}'|T|K^0\rangle^* \tag{4.10}$$

where δ is the scattering phase shift for $|f\rangle$ and $|\bar{f}'\rangle$. Thus we can define the following quantities:

$$\langle 0|T|K^0\rangle = i\exp(i\delta_0)A_0 \tag{4.11a}$$

$$\langle 0|T|\overline{K^0}\rangle = -i\exp(i\delta_0)A_0^* \tag{4.11b}$$

$$\langle 2|T|K^0\rangle = i\exp(i\delta_2)A_2 \tag{4.11c}$$

$$\langle 2|T|\overline{K^0}\rangle = -i\exp(i\delta_2)A_2 \tag{4.11d}$$

In equations (4.11a)–(4.11d), δ_0 and δ_2 are the *S*-wave scattering phases for $I = 0$ and 2 states, respectively, at the energy of the kaon mass. In these equations it is always possible to choose a phase convention by redefining $|K^0\rangle$ and $|\overline{K^0}\rangle$ so that $A_0 = A_0^* =$ real. In this convention [often called the Wu-Yang convention; see also, Oakes (1968)] the theoretical parameters defined in equations (4.2a)–(4.2c) can be expressed with the aid of equations (2.5) and (4.11a)–(4.11d) as

$$\epsilon = \frac{p - q(A_0^*/A_0)}{p + q(A_0^*/A_0)} = \frac{p-q}{p+q} \tag{4.12}$$

$$\epsilon' = \frac{1}{\sqrt{2}}\frac{pA_2 - qA_2^*}{pA_0 + qA_0^*}\exp[i(\delta_2 - \delta_0)] \tag{4.13a}$$

$$\epsilon' \simeq \frac{i}{\sqrt{2}}\exp[i(\delta_2 - \delta_0)]\frac{\operatorname{Im} A_2}{A_0} \tag{4.13b}$$

$$\omega = \frac{1}{\sqrt{2}}\frac{pA_2 + qA_2^*}{pA_0 + qA_0^*}\exp[i(\delta_2 - \delta_0)] \tag{4.14a}$$

$$\omega \simeq \frac{1}{\sqrt{2}}\exp[i(\delta_2 - \delta_0)]\frac{\operatorname{Re} A_2}{A_0} \tag{4.14b}$$

In obtaining equations (4.13b) and (4.14b) we used the fact that $|p| \simeq |q|$. This may be a reasonable approximation due to the fact that the *CP*-nonconserving overlap α defined in equation (2.12) is experimentally of the order of 10^{-3}, as will be discussed in a later part of this section.

It may be worthwhile to point out that if $A_0 = \text{real}$ and $A_2 = \text{real}$† also (i.e. in the case of T invariance) equations (4.13a) and (4.14a) reduce to

$$\epsilon' = \frac{1}{\sqrt{2}}\frac{p-q}{p+q}\frac{A_2}{A_0}\exp[i(\delta_2 - \delta_0)] \tag{4.15}$$

$$\omega = \frac{1}{\sqrt{2}}\frac{A_2}{A_0}\exp[i(\delta_2 - \delta_0)] \tag{4.16}$$

Thus under these circumstances,

$$\epsilon' = \epsilon\omega \tag{4.17}$$

and this in turn, in view of equations (4.5a) and (4.5b) gives the relation‡

$$\eta_{+-} = \eta_{00} = \epsilon \tag{4.18}$$

By equation (4.12), q may be expressed as

$$q = \frac{1-\epsilon}{1+\epsilon}p \tag{4.19}$$

The *CP*-nonconserving overlap α defined in equation (2.12) can be related to ϵ by means of equation (4.19),

$$\begin{aligned}\alpha &= \langle K_L | K_S \rangle = |p|^2 - |q|^2 \\ &= |p|^2\left(1 - \left|\frac{1-\epsilon}{1+\epsilon}\right|^2\right) \\ &= 2\,\mathrm{Re}\,\epsilon(1 - |\epsilon|^2) \\ &\simeq 2\,\mathrm{Re}\,\epsilon\end{aligned} \tag{4.20}$$

Here we have neglected the term of the order of ϵ^3.

The unitarity relation in equation (4.3d) can now be expressed in terms of ϵ by equation (4.20).

$$\begin{aligned}&\langle 0|T|K_L\rangle^* \langle 0|T|K_S\rangle + \langle 2|T|K_L\rangle^* \langle 2|T|K_S\rangle \\ &\simeq -i\left(\Delta m + \frac{i}{2}\Gamma_S\right)\alpha \\ &\simeq -i\left(\Delta m + \frac{i}{2}\Gamma_S\right)2\,\mathrm{Re}\,\epsilon\end{aligned} \tag{4.21}$$

where Γ_L is neglected because $\Gamma_L \ll \Gamma_S$, and

$$\Delta m = m_L - m_S \tag{4.22}$$

† $|\mathrm{Im}(A_2/A_0)| \leqslant 3 \times 10^{-3}$. See Abbud, F., Lee, B. W. and Yang, C. N. (1967). *Physical Review Letters*, **18**, 980.

‡ This relation is the same as in the case of superweak model discussed in Section 7.

If we neglect the $\Delta I = \frac{3}{2}$ transition and only $\Delta I = \frac{1}{2}$ is considered (Olesen, 1967), equation (4.21) becomes

$$-i\left(\Delta m + \frac{i}{2}\Gamma_S\right) 2\,\mathrm{Re}\,\epsilon \simeq \epsilon^* \Gamma_S \tag{4.23}$$

by virtue of the definitions of ϵ and Γ_S. The real part of equation (4.23) gives a trivial identity $\Gamma_S = \Gamma_S$, but the imaginary part provides the relation

$$\frac{\mathrm{Im}\,\epsilon}{\mathrm{Re}\,\epsilon} \simeq \frac{2\Delta m}{\Gamma_S} \tag{4.24}$$

This determines the phase of ϵ in terms of Δm and Γ_S for the 2π decays with only $\Delta I = \frac{1}{2}$.

The real part of ϵ can be determined by considering $K_L \to \pi^\pm e^\mp \nu$ decays. Although there is reasonable evidence that the $\Delta S = \Delta Q$ (S = strangeness and Q = electric charge) rule holds for strangeness-changing semileptonic modes, experimental results are also consistent with the small (<20%) violation of the $\Delta S = \Delta Q$ rule for the neutral K_{l3} decays. Allowing such a possibility and assuming *CPT* invariance, we define the four semileptonic decay amplitudes (Olesen, 1967; Lee, 1966) as

$$\langle \pi^- l^+ \nu | T | K^0 \rangle = f, \qquad \frac{\Delta S}{\Delta Q} = +1, \qquad \Delta I = \tfrac{1}{2}, \tfrac{3}{2} \tag{4.25a}$$

$$\langle \pi^+ l^- \nu | T | \overline{K^0} \rangle = f^*, \qquad \frac{\Delta S}{\Delta Q} = +1, \qquad \Delta I = \tfrac{1}{2}, \tfrac{3}{2} \tag{4.25b}$$

$$\langle \pi^- l^+ \nu | T | \overline{K^0} \rangle = g, \qquad \frac{\Delta S}{\Delta Q} = -1, \qquad \Delta I = \tfrac{3}{2} \tag{4.25c}$$

$$\langle \pi^+ l^- \nu | T | K^0 \rangle = g^*, \qquad \frac{\Delta S}{\Delta Q} = -1, \qquad \Delta I = \tfrac{3}{2} \tag{4.25d}$$

We note that the small violation of the $\Delta S = \Delta Q$ rule may be related to the smallness of the $\Delta I = \frac{3}{2}$ contribution to the amplitudes.

Let the $\Delta S = \Delta Q$ violation parameter† X be

$$X \equiv \frac{g}{f} \tag{4.26}$$

where $X = 0$ corresponds to the $\Delta S = \Delta Q$ rule for weak interaction. If *CP* invariance holds, $f = f^*$, $g = g^*$, and thus $\mathrm{Im}\,X = 0$. If *CP* invariance is violated then, in general, $\mathrm{Im}\,X \neq 0$. It is to be noted that the $\Delta S/\Delta Q = +1$ part and the $\Delta S/\Delta Q = -1$ part may contain respectively both *CP*-invariant and *CP*-noninvariant amplitudes.

We define the charge asymmetry parameter for $K_L \to \pi l \nu$,

$$\delta \equiv \frac{\Gamma_+ - \Gamma_-}{\Gamma_+ + \Gamma_-} \tag{4.27a}$$

† It is known that $\mathrm{Im}\,X = -0{\cdot}08 \pm 0{\cdot}08$ and $\mathrm{Re}\,X = 0{\cdot}22 {}^{+0{\cdot}07}_{-0{\cdot}09}$. See Webber, B. R. (1968). *Physical Review Letters*, **21**, 498; or Olesen (1967).

where

$$\Gamma_{\pm} \equiv \Gamma(K_L \to \pi^{\mp} l^{\pm} \nu) \tag{4.27b}$$

Expressing the amplitudes in terms of f and g defined in equations (4.25a)–(4.25d), δ may be written as

$$\delta \simeq \frac{\alpha(1-|X|^2)}{|1+X|^2} \tag{4.28a}$$

or

$$\delta \simeq \frac{\alpha(1-|X|^2)}{1+2\,\mathrm{Re}\,X} \tag{4.28b}$$

In deriving equations (4.28a) and (4.28b) we used the fact that $p \simeq \frac{1}{2}$ and $|X| \leqslant 0{\cdot}2$. If we assume the $\Delta S = \Delta Q$ rule, equation (4.28b) will be

$$\delta \simeq \alpha \simeq 2\,\mathrm{Re}\,\epsilon \tag{4.29}$$

by equation (4.20).

TABLE 1. Comparison of the results of the superweak model and the scalar meson dominance model with experiment.

Experiment		References	Superweak model	Scalar meson dominance model
$\lvert\eta_{+-}\rvert \times 10^3$	$1{\cdot}89 \pm 0{\cdot}09$	*a*	Input	Input
ϕ_{+-}	$51{\cdot}2° \pm 11°$	*b*	$43{\cdot}8°$	$53{\cdot}6°$
$\lvert\eta_{00}\rvert \times 10^3$	$2{\cdot}3 \pm 0{\cdot}3$	*c*	$1{\cdot}89$	$2{\cdot}32$
$\Delta m/\Gamma_S$	$0{\cdot}48 \pm 0{\cdot}02$	*a*	—	—
$\mathrm{Re}\,\epsilon \times 10^3$	$1{\cdot}12 \pm 0{\cdot}18$	*d*	$1{\cdot}36$	Input
ϕ_{00}	—		$43{\cdot}8°$	$61{\cdot}1°$
μ^+/Z	—		—	$5{\cdot}28 \times 10^{-3}$
g_γ	—		—	$0{\cdot}158\mu^+$
$f'_{\Delta I=1/2}$	—		—	$9{\cdot}72 \times 10^{-9}$

[a] Rosenfeld, A. H. *et al.* (1968). *Review of Modern Physics*, **40**, 77.
[b] Bennet, S. *et al.* (1968). *Physics Letters*, **27B**, 248.
[c] Banner, M. *et al.* (1968). *Physical Review Letters*, **21**, 1107.
[d] Bennet, S. *et al.* (1967). *Physical Review Letters*, **19**, 993.

The experimental values of various parameters are shown in Table 1. With the values given in Table 1,

$$\mathrm{Re}\,\epsilon \cong \frac{\delta}{2} = 1{\cdot}12 \times 10^{-3} \tag{4.30a}$$

$$\frac{\mathrm{Im}\,\epsilon}{\mathrm{Re}\,\epsilon} \simeq \frac{2\Delta m}{\Gamma_S} = 0{\cdot}96 \tag{4.30b}$$

This gives $\text{Im}\,\epsilon = 1{\cdot}08 \times 10^{-3}$, $|\epsilon| = 1{\cdot}55 \times 10^{-3}$, or the phase of ϵ, $\phi_\epsilon = 43{\cdot}8°$. We note that equation (4.7c) provides

$$\cos\phi_{00} \simeq \frac{3\,\text{Re}\,\epsilon - 2\,\text{Re}\,\eta_{+-}}{|\eta_{00}|} \tag{4.31a}$$

i.e.

$$\phi_{00} \simeq \begin{cases} 67{\cdot}0°, & \text{for } |\eta_{+-}| = 1{\cdot}96 \times 10^{-3} \\ 64{\cdot}6°, & \text{for } |\eta_{+-}| = 1{\cdot}89 \times 10^{-3} \end{cases} \tag{4.31b}$$

5. *Electric Dipole Moment of the Neutron*

We now discuss the constraints (Nishijima, 1964; Roman, 1961) imposed by the smallness of the electric dipole moment of the neutron and the observation of *CP* violation.

Let the total Hamiltonian of the neutron be

$$H_t = H_1(P = +1) + FH_2(P = -1) \tag{5.1}$$

Here the parity-even part is H_1, the parity-odd part is H_2, and F is a dimensionless constant. If the neutron has non-vanishing electric dipole moment μ, then in the presence of the external electric field $\vec{E}$, the expectation value of the H_t for the neutron must contain a term corresponding to the potential energy of the dipole moment proportional to $\vec{E}$. Since the direction of the dipole moment can only be associated with spin $\vec{\sigma}$ of the neutron, the interaction corresponding to the dipole moment must be of the type $\vec{\sigma}\cdot\vec{E}$. The spin $\vec{\sigma}$ is odd under time reversal and even under parity, while the electric field $\vec{E}$ is even under T and odd under P. Thus the interaction due to dipole moment is a pseudoscalar and is not invariant under P and T, unless the dipole moment $\mu = 0$. We note that the electric dipole moment μ is

$$\mu \sim Fe\lambda\!\!\!^{-} \tag{5.2}$$

where e is the electronic charge and $\lambda\!\!\!^{-}$ is the dimension of the neutron.

The present limit (Purcell & Ramsey, 1950; Smith *et al.*, 1957) of the electric dipole moment of the neutron is

$$\frac{\mu}{e} \simeq 10^{-22}\ \text{cm} \tag{5.3}$$

The smallness of μ may justify us to assume that $\mu = 0$. In the case of $\Delta S = 0$ we have the following choices imposed:

(i) H_t is parity invariant, and violates C and T invariance, so that *CP* violation results;
(ii) H_t is T invariant, and violates C and P invariance, so that *CP* conservation holds.

In view of the observed *CP* violation in the neutral *K* decays, therefore, the first choice given above seems to be the one we need for $\Delta S = 0$ if $\mu = 0$. Note that the effective $\vec{\sigma} \cdot \vec{E}$ type of interaction may be given by any $H_2(P = -1)$ interaction including weak interactions (H_W). We also note that the above argument is true for the nucleon or any stable particle.

6. *η Asymmetry and C Noninvariance in* $\eta_{3\pi}$

It was suggested that observation of an asymmetry, i.e., the difference between the π^+ and π^- energy spectra in $\eta \to \pi^+\pi^-\pi^0$ is a proof of a *C*-noninvariant electromagnetic (EM) interaction of the hadron. Experiment (Gormley *et al.*, 1968) shows that there may exist such a small asymmetry. The reaction they observed is

$$\pi^- p \to \pi^+ \pi^- \pi^0 n \tag{6.1}$$

The asymmetry *A* is found to be

$$A \equiv \frac{N^+ - N^-}{N^+ + N^-} = 1{\cdot}5 \pm 0{\cdot}5\% \tag{6.2}$$

where $N^+ \equiv$ number of events for which π^+ energy $> \pi^-$ energy,

$N^- \equiv$ number of events for which π^- energy $> \pi^+$ energy.

The result given in equation (6.2) is supposedly found by subtracting the effect of a possible background.

However, it was shown (Yuta & Okubo, 1968) that a small charge asymmetry can be produced without *C* nonconservation but by interference effect with the background, by assuming that *C* invariance holds for the $\eta_{3\pi}$ decay

$$\begin{array}{l}\pi^- p \to n\eta \\ \qquad\quad \hookrightarrow \pi^+\pi^-\pi^0\end{array} \tag{6.3}$$

and the charge asymmetry for the 3π background

$$\pi^- p \to n\pi^+\pi^-\pi^0 \tag{6.4}$$

is negligible. The maximum charge asymmetry is found as

$$A_{\max} = 1{\cdot}6\% \tag{6.5}$$

If this is correct the experimental asymmetry given in equation (6.2) may be due to the interference of η and 3π background, not necessarily by *C* nonconservation. If this is actually the case, $A_{\exp}$ should be dependent on the incident pion energy as well as production mechanism. So to identify $A_{\exp}$ with a *C* nonconservation in η decay, $A_{\max}$ has to be made as small as possible by changing the incident energy and the production mechanism. This is not done yet. However, it may be reasonable to expect that *A* may be smaller than the value shown in equation (6.2).

7. Theoretical Models

Theoretically any model describing the apparent CP violation in $K_L \to 2\pi$ decays has to be consistent with the following general constraints, as we have discussed in the previous sections.

(i) The interaction which causes the CP violation has to conserve P, and violate C and T (due to the smallness of the electric dipole moment of the neutron), if the interaction has $\Delta S = 0$ (see Section 5).
(ii) If the interaction which causes CP violation is not $\Delta S = 0$, but $\Delta S = \pm 1$ then the $\Delta S = 0$ part of interaction must be T conserving and violate C and P (see Section 5).
(iii) The CP violation has to be small, of the order of 10^{-3}, compared with the CP-conserving part of the amplitudes.
(iv) Only $K_L \to 2\pi$ decays exhibit the observable effect of the CP violation.

In addition to these constraints, of course, any model has to predict the experimental parameters shown in Table 1.

Since there is only one observed case for CP-violation, and

$$[H_t, CP] \neq 0, \qquad H_t = H_S + H_\gamma + H_W \tag{7.1}$$

there are many possible models, namely, CP-violation caused by either

(i) a new interaction,
(ii) electromagnetic interaction, weak, or strong interaction,
(iii) combinations of these.

We discuss some of the models in the following.

(*a*) *Superweak Model*

Since only $K_L \to 2\pi$ exhibits any observable CP violation, a model has to have only a few CP-violating parameters, preferably one or two. A model with one parameter is the superweak model (Lee & Wolfenstein, 1965).

This model assumes that all CP-violating phenomena are due to the CP noninvariance of the mass operator, not due to the decay part of the matrix element. We assume that the Hamiltonian consists of two parts,

$$H = H_W(|\Delta Y| \neq 2) + H_{SW}(|\Delta Y| = 2) \tag{7.2}$$

where

$$\langle \overline{K^0} | H_{SW} | K^0 \rangle \neq 0 \tag{7.3}$$

In equation (7.2), H_W is the CP-invariant weak interaction and H_{SW} is the CP-violating superweak interaction. As shown in equations (4.12)–(4.14b), the theoretical parameter ϵ is independent of the decay amplitudes A_0 and A_2, and ϵ' and ω are dependent on the decay amplitudes.

By equations (2.5), (2.6) and (4.12), $|K_S\rangle$ and $|K_L\rangle$ may be expressed in terms of ϵ only,

$$|K_S\rangle = \frac{1}{\sqrt{2}} \frac{1}{(1+|\epsilon|^2)^{1/2}} [(1+\epsilon)|K^0\rangle - (1-\epsilon)|\overline{K^0}\rangle] \tag{7.4a}$$

$$|K_L\rangle = \frac{1}{\sqrt{2}} \frac{1}{(1+|\epsilon|^2)^{1/2}} [(1+\epsilon)|K^0\rangle + (1-\epsilon)|\overline{K^0}\rangle] \tag{7.4b}$$

Thus CP violation comes only from the states, $|K_S\rangle$ and $|K_L\rangle$. The superweak model depends on only one parameter ϵ which does not depend on the decay part but on the mass part alone. By equations (4.5a) and (4.5b), therefore, this model predicts

$$\eta_{+-} = \eta_{00} = \epsilon \tag{7.5}$$

This result is the same as for the case when we neglect $\Delta I = \frac{3}{2}$, as shown in equation (4.8), and the case when both A_0 and A_2 are real, as given in equation (4.18). So equation (7.5) is by no means an unique result of this model.

By the unitarity relation, we get from equation (4.30b)

$$\phi_{+-} = \phi_{00} = \tan^{-1} \frac{\text{Im}\,\epsilon}{\text{Re}\,\epsilon} \simeq \tan^{-1}\left(\frac{2\Delta m}{\Gamma_S}\right) \tag{7.6}$$

These predictions of this model [equations (7.5) and (7.6)] are shown in the third column of Table 1, and are not inconsistent with experiment.

To estimate roughly the relative strength of $H_{SW}(aG)$ and $H_W(G)$, we consider the CP-invariant self-energy term,

$$\Delta_W = \sum_n \frac{\langle K^0|H_W|n\rangle\langle n|H_W|\overline{K^0}\rangle}{E_K - E_n} \tag{7.7}$$

and the CP-violating contribution of the order aG to the self-energy,

$$\Delta_{SW} = \langle \overline{K^0}|H_{SW}|K^0\rangle \tag{7.8}$$

Thus the ratio of equation (7.8) to equation (7.7) is

$$\frac{\Delta_{SW}}{\Delta_W} \simeq \frac{aG}{G^2 M^2} \simeq 10^5 a \tag{7.9}$$

where the Fermi constant $G \approx 10^{-5}$ M^{-2}. The relative strength a can be estimated by the fact that $\Delta_{SW}/\Delta_W \simeq 10^{-3}$. Thus the superweak interaction is 10^{-8} times weaker than the weak interaction, i.e.,

$$a \simeq 10^{-8} \tag{7.10}$$

(*b*) *Oakes' Model*

This model (Oakes, 1968) assumes a current × current weak interaction constructed from the neutral vector and axial-vector current in addition to the charged current introduced by Cabibbo. It also assumes conservation of the vector current (CVC), universality, lepton conservation, and *CPT* invariance.

The weak interaction so constructed is

$$H_W = \frac{G}{\sqrt{2}} [\tfrac{1}{2}\{J^{(+)}, J^{(-)}\} + J^{(0)}J^{(0)}] \tag{7.11}$$

where

$$J_\lambda^{(-)} = \cos\theta (V-A)_\lambda^{(1-2i)} + \sin\theta (V-A)_\lambda^{(4-5i)} + l_\gamma^{(-)} \tag{7.12a}$$

$$l_\lambda^{(-)} = \bar{\mu}\nu_\lambda (1-\gamma_5)\nu_\mu + \bar{e}\gamma_\mu (1-\gamma_5)\nu_e \tag{7.12b}$$

$$J^{(+)} = J^{(-)\dagger} \tag{7.12c}$$

$$J_\lambda^{(0)} = \cos\phi (V+A)_\lambda^{(3-\sqrt{3}\cdot 8)} + i\sin\phi (V+A)_\lambda^{(6-i7)} - i\sin\phi (V+A)_\lambda^{(6-i7)} + l_\lambda^{(0)} \tag{7.12d}$$

$$l_\lambda^{(0)} = \bar{\nu}_\mu \gamma_\lambda (1+\gamma_5)\nu_\mu + \bar{\nu}_e \gamma_\lambda (1+\gamma_5)\nu_e \tag{7.12e}$$

In equation (7.11), the neutral part $J^{(0)}J^{(0)}$ violates *CP* invariance but assures *CPT* invariance. To the lowest order, the $\Delta S = 0$ nonleptonic part is *CP* invariant (*T* invariant). Thus, according to the arguments in Section 5, the electric dipole moment of the neutron is zero. To the lowest order, only the $\Delta S = \pm 1$ nonleptonic part is *CP* nonconserving. This model includes the *CP*-violating effect from the mass matrix also, and gives *CP*-violating effects to most processes but too small to be detected. In hyperon decay, $\Sigma_+^{\ +}$ has a large effect in the asymmetry parameter. But experiment is not accurate enough to test the model. It is also expected that there will be no large effect to asymmetry in $\eta_{3\pi}$ decay.

The predictions on the *CP*-violating parameters are

$$\frac{\eta_{00}}{\eta_{+-}} \sim 1 \pm 3 \left|\frac{\epsilon'}{\epsilon}\right| \exp\{i[\delta_2 - \delta_0 + (\pi/4)]\} \tag{7.13a}$$

$$|\phi| \sim 10^{-3} \text{ (estimated)} \tag{7.13b}$$

The second term in equation (7.13a) is approximately 25%. The Cabibbo angle ϕ belonging to the neutral current turns out to be so much smaller than θ that we cannot escape a strange feeling as to why this is so.

(*c*) *Scalar Meson Dominance Model*

The apparent success of vector-meson dominance in the *CP*-conserving two-pion decays of *K* mesons makes us ponder over the possibility that an effective *CP*-violating interaction may also be developed in a similar fashion (Yun, 1969).† Consider the *CP*-nonconserving interaction for $K_{2\pi}$ decays of the type,

$$H = H_W^{(+)}(CP=+1) + H^{(-)}(CP=-1) \tag{7.14}$$

† For another type of *C*-noninvariant EMI, see Lee, T. D. (1965). *Physical Review*, **140**, 959.

where $H_W^{(+)}$ is the CP-even weak interaction and $H^{(-)}$ is the CP-odd interaction. We assume that the CP-even part of the amplitudes is dominated by vector mesons, while the CP-odd part is dominated by the observed physical scalar mesons octet $[\pi_V(1016), \eta_V(1070), K_V(1080)]$ as shown in Fig. 1.

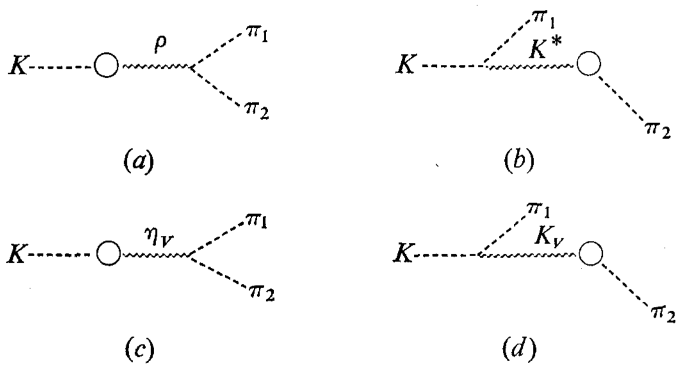

Figure 1.—(*a*) and (*b*) are the diagrams for the CP-conserving part, and (*c*) and (*d*) are for the CP-violating part of the $K_{2\pi}$ decays.

The relevant Hamiltonians are

$$H_W^{(+)} = f\{V_\mu, \partial_\mu P\} \tag{7.15a}$$

$$H^{(-)} = f'\{\partial_\mu S, \partial_\mu P\} \tag{7.15b}$$

$$H_{VPP} = \tfrac{1}{2} g_{\rho\pi\pi}\, \boldsymbol{\rho}_\mu \cdot \boldsymbol{\pi} \times \overleftrightarrow{\partial}_\mu \pi + i g_{K^*K\pi} K_\mu^{*}\dagger\, \boldsymbol{\tau} K \cdot \overleftrightarrow{\partial}_\mu \pi \tag{7.15c}$$

$$H_{SPP} = g_{\eta_V \pi\pi}\, \eta_V \boldsymbol{\pi} \cdot \boldsymbol{\pi} + g_{KK_V\pi} K_V \dagger\, \boldsymbol{\tau} K \cdot \boldsymbol{\pi} \tag{7.15d}$$

The decay amplitudes of K_S and K_L may be written as

$$\langle 2\pi | H | K_S \rangle = \frac{(p+q)}{\sqrt{2}} [i \langle 2\pi | H_W^{(+)} | K_1 \rangle + \bar{\epsilon} \langle 2\pi | H^{(-)} | K_2 \rangle] \tag{7.16a}$$

$$\langle 2\pi | H | K_L \rangle = \frac{(p+q)}{\sqrt{2}} [\langle 2\pi | H^{(-)} | K_2 \rangle + i\bar{\epsilon} \langle 2\pi | H_W^{(+)} | K_1 \rangle] \tag{7.16b}$$

by equations (2.5) and (7.14), and

$$\bar{\epsilon} \equiv \frac{p-q}{p+q} \tag{7.16c}$$

Note that in this model the phase convention is different from that of Wu and Yang;

$$\langle 0 | H_W^{(+)} | K^0 \rangle = i(\operatorname{Re} A_0) \exp(i\delta_0) \tag{7.17a}$$

$$\langle 0 | H^{(-)} | K^0 \rangle = i(i \operatorname{Im} A_0) \exp(i\delta_0) \tag{7.17b}$$

and similarly for $I=2$ final states, while Wu and Yang's convention is

$$\langle 0|H|K^0\rangle = iA_0 \exp(i\delta_0) \tag{7.18a}$$

$$\langle 0|H|\overline{K^0}\rangle = -iA_0 \exp(i\delta_0) \tag{7.18b}$$

Thus in this model, $\epsilon \neq \bar{\epsilon}$ as in equation (4.12) but

$$\epsilon = \bar{\epsilon} - i\frac{\langle 0|H^{(-)}|K_2\rangle}{\langle 0|H^{(+)}|K_1\rangle} \tag{7.19}$$

The difference in the phase convention can be seen by the amplitudes

$$\langle \pi^+\pi^-|H_W^{(+)}|K_1\rangle = -(g_{\rho\pi\pi} f^{\Delta I=1/2})\frac{K^{02}-\mu^{+2}}{K^{*2}}\left(1-\frac{a}{2}\right) \tag{7.20a}$$

$$\langle \pi^+\pi^-|H^{(-)}|K_2\rangle = \frac{1}{\sqrt{3}}(g_{\pi_V\pi\eta} f'^{\Delta I=1/2})\left[\frac{K^{02}}{\eta_V{}^2-K^{02}} - \frac{3\mu^{+2}}{K_V{}^2-\mu^{+2}}(1+b)\right] \tag{7.20b}$$

where

$$a \equiv \frac{f^{\Delta I=3/2}}{f^{\Delta I=1/2}} \tag{7.20c}$$

$$b \equiv \frac{f'^{\Delta I=3/2}}{f'^{\Delta I=1/2}} \tag{7.20d}$$

The $K^+ \to \pi^+\pi^0$ and $K_S \to \pi^+\pi^0$ data provides us with

$$|a| = 3{\cdot}1 \times 10^{-2} \tag{7.21a}$$

$$|f^{\Delta I=1/2}| = 1{\cdot}84 \times 10^{-6}\mu^+ \tag{7.21b}$$

We now assume that $H^{(-)}$ consists of two factors,

$$H^{(-)} = H_\gamma^{(-)} H_W^{(+)} \tag{7.22a}$$

and

$$H_\gamma^{(-)} = g_\gamma \partial_\mu \pi_V \rho_\mu \tag{7.22b}$$

where $H_\gamma^{(-)}$ is the effective *CP*-violating second-order electromagnetic interaction (EMI) as shown in Fig. 2. This enables us to express f' as

$$f'^{\Delta I=1/2,\,3/2} = \frac{1}{\rho^{+2}} g^{\Delta I=0,\,2} f^{\Delta I=1/2,\,3/2} \tag{7.23}$$

If we assume that the $\Delta I=0$ part is dominant over the $\Delta I=2$ belonging to 27, we get

$$|a| \simeq |b| \tag{7.24}$$

and

$$\eta_{+-} \simeq \bar{\epsilon} - i\frac{\langle \pi^+\pi^-|H^{(-)}|K_2\rangle}{\langle \pi^+\pi^-|H_W^{(+)}|K_1\rangle} \tag{7.25a}$$

$$\eta_{00} \simeq \bar{\epsilon} - i \frac{\langle \pi^0 \pi^0 | H^{(-)} | K_2 \rangle}{\langle \pi^0 \pi^0 | H_W^{(+)} | K_1 \rangle} \tag{7.25b}$$

The predictions of this model are shown in the last column of Table 1. It is interesting to note that

$$Z \equiv \frac{f^{\Delta I = 1/2}}{f'^{\Delta I = 1/2}} \simeq \alpha \text{ (fine structure constant) } \mu^{+}$$

and g_γ has almost the same magnitude as for the *CP*-conserving effective second-order electromagnetic coupling constant in the two-pole model of $\eta_{3\pi}$ decays. The *CP*-violating contribution to the self-energy is due to the term $H_W^{(+)} H^{(-)}$. $\eta_{3\pi}$ decay amplitudes and $\eta_{3\pi}$ charge asymmetry are expected to be small.

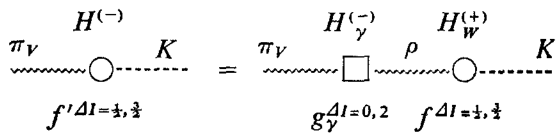

Figure 2.—The effective *CP*-violating interaction in terms of the *CP*-conserving weak interaction and the effective *CP*-violating second-order EMI.

If the exact unitarity relation including $\Delta I = \frac{3}{2}$ in the 2π modes,

$$\frac{\text{Im}\,(\eta_{+-} + \eta_{00} R)}{\text{Re}\,(\eta_{+-} + \eta_{00} R)} = \frac{2\Delta m}{\Gamma_S} = 0{\cdot}96$$

where R is given by equation (4.3c), is used instead of equation (4.24), $|\eta_{00}|$ and ϕ_{00} become lower than the values given in the last column of Table 1 while ϕ_{+-} remains the same.

(*d*) *Nishijima's Model*

This theory (Nishijima, 1967) of weak interaction is different from the current times current type given by Cabibbo. It employs the *S*-matrix description of the transition amplitudes and postulates an unsubtracted dispersion relation for all weak vertex functions. By translating the unsubtracted dispersion relation into the language of field theory, it provides a self-consistency condition and derives the Goldberger-Treiman relation, Gell-Mann-Okubo mass formula, and Coleman-Glashow mass formula.

The nonleptonic and semileptonic Hamiltonians are of the form,

$$H_{NL}(x) = i \int_{y_0 = x_0} d^3y [K_0(y), H_S(x)] \tag{7.26a}$$

in the Heisenberg representation and

$$H_{\text{lept}}(x) = -\mathcal{J}_\lambda(x) j_\lambda{}^\dagger(x) - \mathcal{J}_\lambda{}^\dagger(x) j_\lambda(x) \tag{7.26b}$$

where the hadronic current is

$$\mathscr{J}_\lambda(x) = \mathscr{J}_\lambda^{\Delta S=0} + i\mathscr{J}_\lambda^{\Delta S=1}(x) \tag{7.26c}$$

In equation (7.26a), K_0 is the neutral part of the hadronic current transforming like $|\Delta Y| = 1$. j_λ in equation (7.26b) is the leptonic current. Note that i in equation (7.26c) indicates *CP*-violation. The enhancement mechanism necessary to introduce the Cabibbo angle leads to a pronounced effect of *CP*-violation and the second-order weak interaction is enhanced to produce $\langle 2\pi|H_W^{(2)}|K_L\rangle$. Then it gives $\epsilon \simeq 10^{-3}$.

In this theory, the problem of *CP*-violation and that of Cabibbo angle seem closely related. Thus solution of one would lead to the other. Another interesting feature is that commutation relations implied by current algebra are produced without reference to the fictitious quarks.

(*e*) *The Three Triplet Model*

This model (Woo, 1968) applies the weak currents in the Han-Nambu model, and with certain assumption of symmetry breaking it produces a maximal *CP*-violation of the type $i\mathscr{J}_\lambda^{\Delta S=1}$. This model gives *CP*-violation of the order of $e^2 G$ in most cases of semileptonic and nonleptonic processes.

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